

## INTERACTIVE PHYSICS & MECHANICAL VIBRATIONS

### THE SIMPLE ANALYSIS OF VEHICLE BODY

A Vehicle has a suspension including spring “k” and damper “c” as shown on the Figure 1 under the normal conditions but to be able make it understandable we will simplify the modelling for university or high school students.

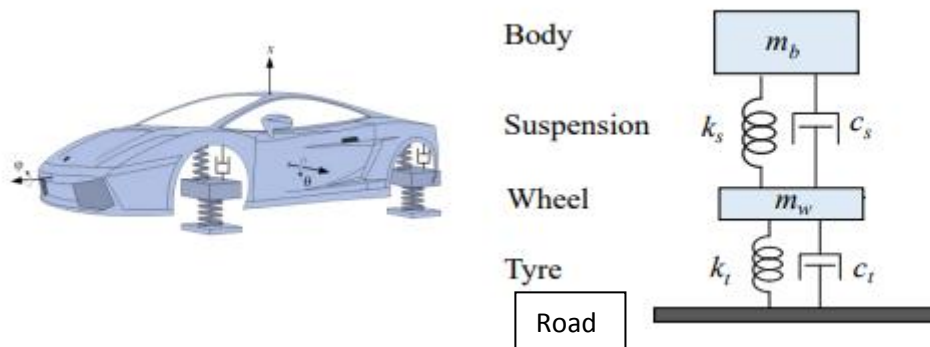


Figure 1

Especially the springs shown in Figure 2, would be based for modelling of mathematical and physical. And also calculated only springs(not dampers) between road and wheels of vehicle as per numbers highlighted with red color below (By Interactive Physics)

Totally 4 Springs(Tyre) for 4 Wheels

One Vehicle Body

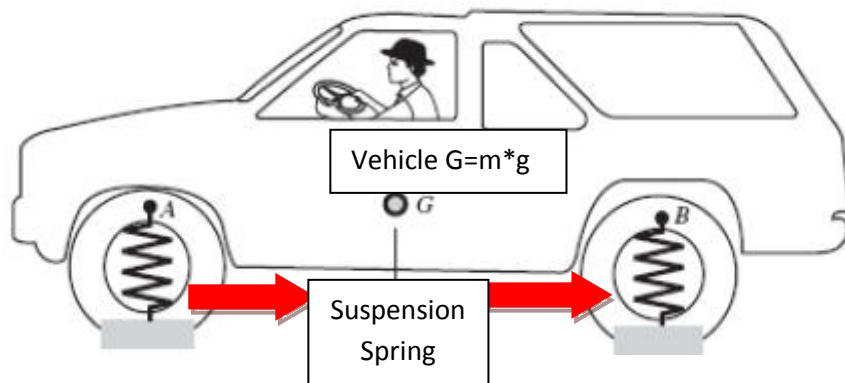


Figure 2

As shown in the figure below, mechanical system has been drawn by interactive physics program. After analysing the motion according to the values of suspension springs' coefficient, mass of vehicle and road friction coefficient specified by Designers and Users of this program, we will create/draw a free body diagram which has the simple single degree of freedom of the vehicle

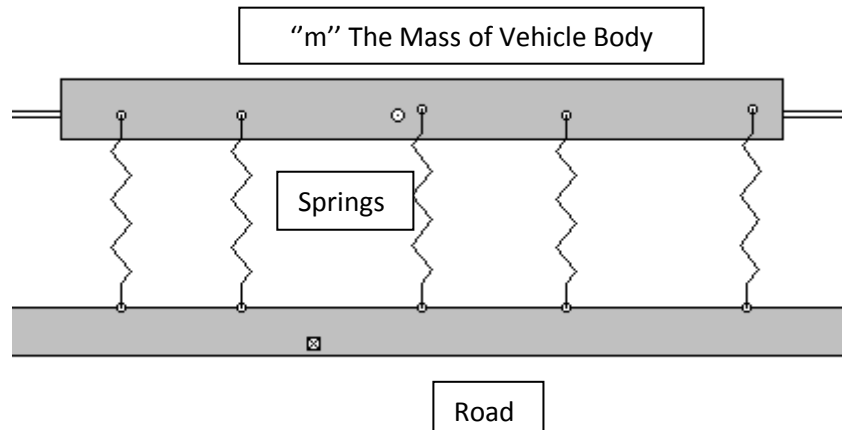


Figure 3

For vehicle body and the modelling of interactive physics above, the deflections of all springs are same and the mass-spring system designed in Figure 4 has a **single degree of freedom system**. (This is an engineering acceptance for us)

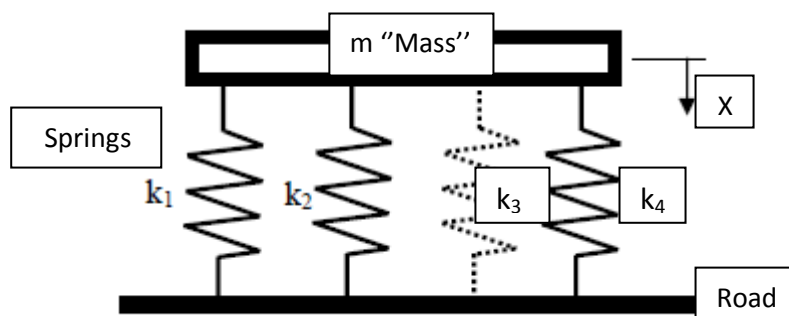


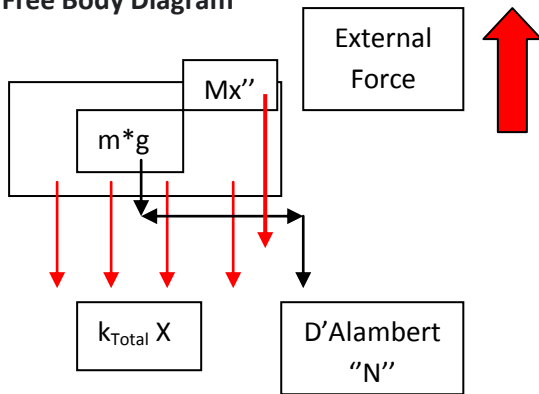
Figure 4

F=Force, m=Mass, a=Acceleration, k=Spring coefficient, x=The value of deflection

$$\sum F = k_1 \cdot x + k_2 \cdot x + k_3 \cdot x + k_4 \cdot x = k_{\text{total}} \cdot x = K_T \cdot x$$

$$k_T = k_1 + k_2 + k_3 + k_4$$

### Free Body Diagram



$$F = m \times a, X = v \times t$$

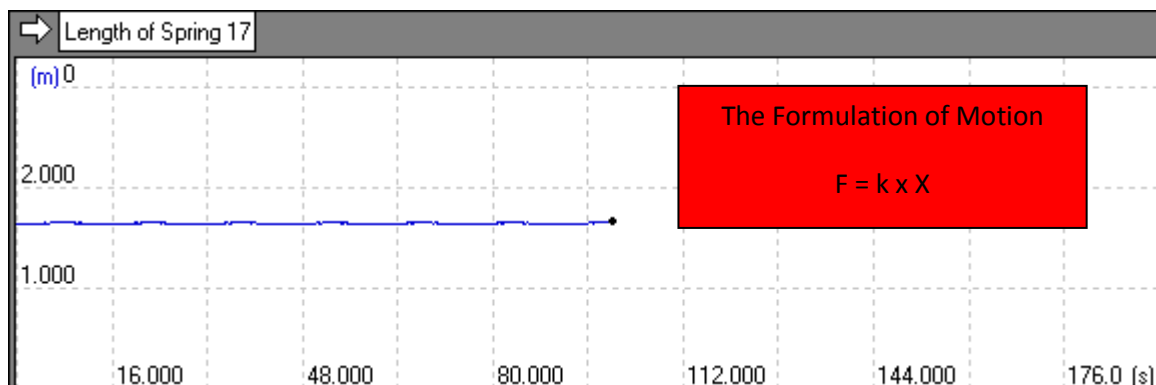
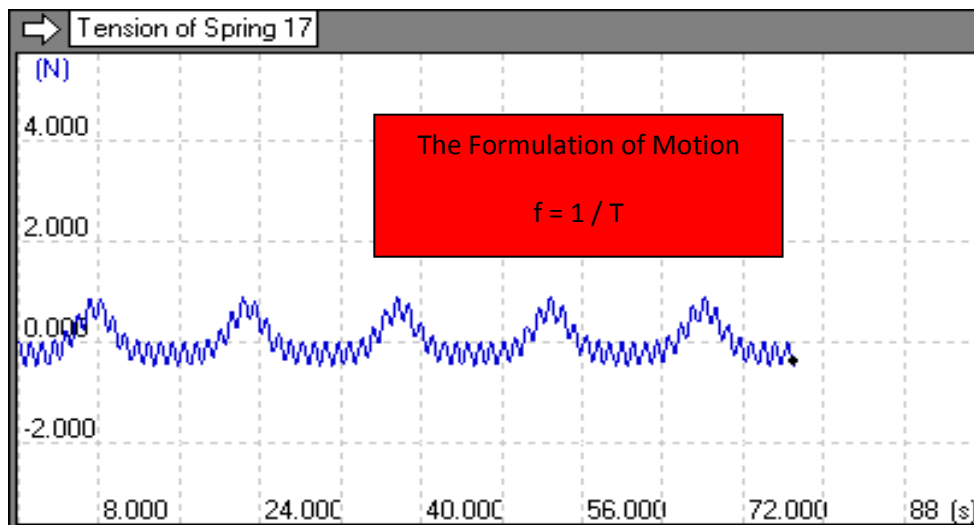
For derivative  $X' = v$  (velocity),  $X'' = a$  (acceleration)

$$Mx'' = -K_T X \ggggg Mx'' + K_T X = 0 \text{ (Equation for Figure 4)}$$

Provided that adding the external force ( positive direction "+F")

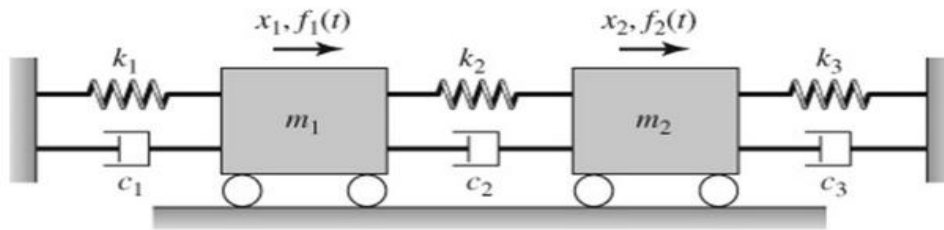
$$F(t) - K_T X = Mx'' \gggggg F(t) = Mx'' + K_T X \text{ (Equation)}$$

### The Motion of Tension and Length of Spring By Interactive Physics

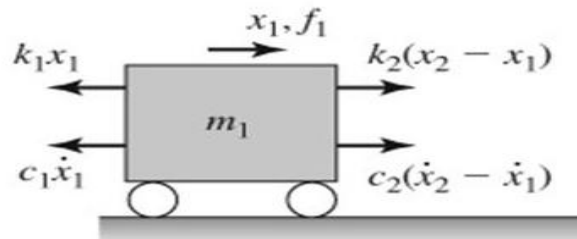


## Multiple Degree of Freedom Vibrating Systems

This exercise By Chris Snook



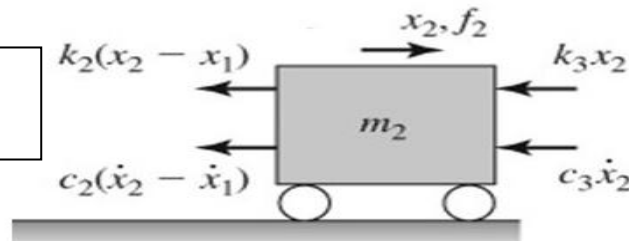
### Newton's 2nd Law



$$-k_1x_1 - c_1\dot{x}_1 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) + f_1(x) = m_1\ddot{x}_1$$

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = f_1(x)$$

Linear Differential Equations



$$m_2\ddot{x}_2 + (c_2 + c_3)\dot{x}_2 - c_2\dot{x}_1 + (k_2 + k_3)x_2 - k_2x_1 = f_2(x)$$

Matrix Form

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & (c_2 + c_3) \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & (k_2 + k_3) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1(x) \\ f_2(x) \end{Bmatrix}$$

### Resources

- <https://yildiz.academia.edu/AhmetOven>
- [https://open.usq.edu.au/pluginfile.php/77992/mod\\_resource/content/3/mec3403/vibration-2/vibration-2.htm](https://open.usq.edu.au/pluginfile.php/77992/mod_resource/content/3/mec3403/vibration-2/vibration-2.htm) /Chris Snook/University of Southern Queensland